A Qualitative Coordinate Language of Location of Figures within the Ground

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Abstract

This paper proposes a qualitative coordinate language of regional individuals. It is shown that representing a configuration of regions using a coordinate language of regional individuals supports a primary breakup of a spatial configuration into primary objects of focus and secondary objects of reference, i.e., in figure and ground. We define the notion of figure and ground in terms of regions and regional partitions using the RCC theory. Coordinates represent the location of a figure region within a partition of ground regions. Coordinates are n-tuples of 'figure ground relations'. In this paper we formally define 'figure ground relations' and analyze their properties. We show that the qualitative coordinate language of regions can be used as a general framework to create finite theories of arbitrary, possibly infinite configurations of regional phenomena.

1 Introduction

Coordinate languages are formal languages that represent individuals in terms of their location within an ordering frame of reference rather than the individuals themselves (Carnap 1958). In this paper we discuss that representing spatial phenomena in a coordinate language corresponds to the way how natural language structures (large scale) space. We show that representing phenomena by means of a coordinate language corresponds to the primary breakup of a scene in primary portions of focus and in secondary portions used to characterize the disposition of the primary portions of focus in natural language (Talmy 1983). In particular we describe how to construct a coordinate language based on a qualitative thing language of regions. Thing languages (Carnap 1958) are languages of individuals rather than languages of location of individuals. In thing languages individuals, relations and operations between them are represented by discrete symbolic terms. The qualitative thing language used is the language of the RCC theory (Randell, Cui et al. 1992).

The construction of the coordinate language is done by distinguishing between single regional phenomena and configurations of regional phenomena forming a partition of space. Single regional phenomena are identified with primary objects of focus, the figures. Configurations of regions forming a partition are identified with the ground and provide a frame of reference used to characterize the disposition of the figure. Coordinates are n-tuples of relations between figure regions and regions belonging to the ground. The base of the coordinate system is provided by the structure of the regional partition representing the ground. We show that for arbitrary finite regional partition a finite coordinate theory of location within this partition can be constructed.

This approach can be applied to different kinds of problems of representing phenomena in geographic space:

- Regional partitions can be used to represent non-bounded, continuously distributed phenomena (Laurini and Thompson 1994). Figure regions can be identified with arbitrary bounded regional phenomena. Coordinate n-tuples represent relations between bounded and continuously distributed phenomena.
- The qualitative coordinate language of regions can be used as a general framework to create finite theories of arbitrary, possibly infinite configurations of regional phenomena.

This paper is organized as follows: Section 2 is a compilation of selected aspects of how natural language structures space based on (Talmy 1983). These aspects are discussed and applied to formal languages, in particular to a logical language of connectedness of regional individuals in the context of its application to represent phenomena in geographic space (Section 3). The construction of the coordinate language is carried out in Sections 4, 5, and 6, by formally defining the notion of location of regional phenomena within a frame of reference provided by a regional partition of the plane. Conclusions are given in Section 7.

2 How (Natural) Language Structures Space

Schematization plays a fundamental role in linguistic space descriptions. It is "a process that involves the systematic selection of certain aspects of a referent scene to represent the whole, while disregarding the remaining aspects" (Talmy 1983, pg. 225). Schematization is a cognitive process, which determines the structure of the core of the system of natural language. The core structures of natural language "… represent only certain categories, such as space, time (hence, also form, location, and motion), … They are not free to express just anything within these conceptual domains, but are limited to quite particular aspects and combinations of aspects, ones that can be thought to constitute the 'structure' of those domains" (Talmy 1983, pg. 228).

The spatial system belongs to the core of natural language. "One main characteristic of language's spatial system is that it imposes a fixed form of structure on virtually every spatial scene … language's system is to work out one portion within a scene for primary focus and to characterize its disposition in terms of a second portion …, and sometimes a third portion …, selected from the remainder of the scene. The primary object's 'disposition' here refers to its site when stationary, its path when moving, and often also its orientation during either state" (Talmy 1983, pg. 229).

The linguistic schemata of focal or primary object and reference or secondary object "are closely related to the notion 'Figure' and 'Ground' described in gestalt psychology and can approximately be applied to them … The figure is a moving or conceptually movable object whose site, path, or orientation is conceived as a variable the particular value of which is the salient issue. The ground is a reference object (itself having a stationary setting within a reference frame) with respect to which the Figure's site, path, or orientation receives characterization" (Talmy 1983, pg. 232).

Language structures reflect the cognitive mode that "our predominant concern is with a smaller portion of focal interest within a broader field and, often also, with a determination of that portion's relation to the field, so that we can achieve direct sensory (or imaginal) contact with it. The very concept of 'location' of an object within a space … owes its existence and character to this cognitive mode. … 'Localizing' an object (determining its location) … involves processes of dividing a space into subregions or segmenting it along its contours, so as to 'narrow in' on an object's immediate environment" (Talmy 1983, pg. 234)

Figures and ground and the location of figures within or with respect to the ground are characterized by language elements referring to the geometry of figure and ground. The geometry of a configuration of phenomena in space is characterized by properties of phenomena and relations between phenomena that remain invariant under classes of transformations (Klein 1872).

The way how natural language structures space is characterized by (Talmy 1983):

- The primary breakup of a spatial scene into primary and secondary portions.
- The characterization of one object's spatial disposition in terms of another, in particular the spatial disposition of a primary object in terms of (a) secondary object(s).
- The correspondence of the distinction between primary and secondary objects in natural language and the distinction between figure and ground in gestalt psychology.
- The characterization of figure and ground in terms of their geometry (rather than, for example, by their material).
- The simpler geometric structure of the figure and the more complex structure of the ground.
- The characterization of the geometry of the figure in terms of its location within the geometry of the ground.

3 Towards A Coordinate Language of Regions

In this section we consider how the basic spatial distinctions made by natural language can be represented in a formal language of the theory of binary topological relations between regional individuals. We carry on the discussion in this section on an intuitive and informal level and postpone the formal treatment to Sections 5 and 6.

Regional individuals are language primitives whose intended interpretations are subspaces in n-dimensional space ($n \geq 1$) which have the same dimension as their embedding space. We assume that the embedding space is compact, i.e., we can find a finite set of regions whose union fills the whole space. In particular, we consider 2-dimensional regions in 2-dimensional Euclidean space (abbreviation - regions in the plane).

In Figure 1 an example for a configuration of regions is given. The discussion in the remainder of this paper is based on this example.

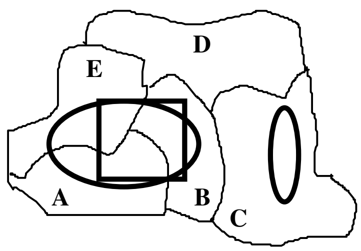

Figure Regions	Ground Regions
R ≡	A
	B
S ≡	C
	D
T ≡	E

Figure 1: A configuration of regions in the plane

3.1 Languages of Binary Topological Relations between Regions

Languages of binary topological relations are formal languages which were designed to represent aspects of the geometry of configurations of regional individuals in the plane which remain invariant under topological transformations. Languages of binary topological relations between regions in the plane are based on a set of jointly exhaustive and pairwise disjoint relations (JEPD). These sets of relations have the property that for arbitrary pairs of regions one and only one relation holds.

Consider the set of binary relations between topologically simple regions in the plane (Figure 2):

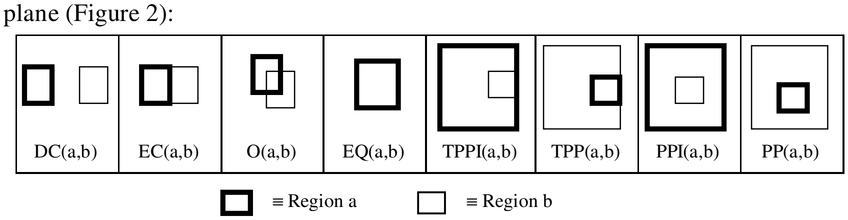

Figure 2: Topological relations between regions in the plane

These relations can be formally defined in different ways, e.g., (Egenhofer and Herring 1990) or (Randell, Cui et al. 1992). In this paper we use formal definitions based on (Randell, Cui et al. 1992).

The configuration of regions in (Figure 1) can be represented in a language of binary relations by the set relations which hold for pairs of individuals of the configuration (Example 1).

```
{EC(A,B),EC(A,E),… ,EC(E,A), DC(A,C),… ,
DC(C,A),O(A,S),O(B,S), O(E,S),…, DC(C,S),…, PP(T,C),…,
PPI(C,T), O(A,R),…,DC(C,R),…, O(R,S),DC(R,T),…, O(S,R)}
```

Example 1: The representation of a configuration of regions in terms of expressions of a language of binary topological relations

Representing a configuration of regions by the set of relations which hold for pairs of individuals implies that none of the structuring distinctions discussed in the context of how natural language structures space (Section 2) are used in this language of relations. The larger the configuration the more relations for pairs of individuals need to be represented explicitly or implicitly. The number of pairs of individuals grows with the power of 2.

3.2 Notion of Figure and Ground in a Formal Language of Regions

Regional Partitions of the Plane

A regional partition of the plane is a set of regions with the following properties:

- No two regions of the partition overlap;
- The union of the regions forming the partition fills the whole plane.

In the remainder we assume finite partitions, i.e., partitions which consist of a finite number of regions. We further exclude the case of a partition consisting of only one, i.e., the universal, region. Consequently partitions consist of `2…n` regions. Finite regional partitions can be constructed from arbitrary finite sets of non-overlapping regions simply by adding the complement region to the set of regions. Region and complement region fill the whole plane and are externally connected. Regions may have holes. Holes may be ‘filled’ with regions. In Figure 1 for example the regions `A,B,C,D,E, complement region of (A+B+C+D+E)` form a finite partition of the plane.

Figure and Ground

Now we apply the way how natural language structures space to a formal language of regional individuals. Figures, i.e., focal or primary objects, are identified with single regions. The regions `R,S,T` in Figure 1 are figure regions. The notion of ground is identified with sets of regional individuals forming a regional partition. In Figure 1 the regions `A,B,C,D,E, complement region of(A+B+C+D+E)` form the ground. Figure regions are located within the frame of reference provided by the ground (Example 2).

```
Consider Figure 1:
•  the figure region T is contained in (a proper part of)
   the ground region C, it does not intersect any
   boundary of a ground region
•  the figure regions R and S are contained in the union
   of the neighboring ground regions A,B,E. R and S cross
   the boundaries between A and B, A and E, and B and E
```

Example 2: Location of figure regions within the ground

Location of (figure) regions in partitions of (ground) regions is compatible with the general properties of figure and ground:

- Single (figure) regions are geometrically simpler than sets of regions forming a regional partition (the ground is a set of regions obeying certain laws);
- Relations between figure and ground are asymmetric since figure and ground are of different types (individuals vs. sets consisting of at least two individuals);
- Figures are located within the ground (Section 6), the partition serves as frame of reference.

Locating (figure) regions in partitions of (ground) regions corresponds to the basic spatial distinctions made by natural language: The primary breakup of a scene in primary portions of focus and in secondary portions used to characterize the disposition of the primary portions of focus.

A Coordinate Language of Regions

"By a coordinate language we understand a language in which the form of an individual expression indicates the position of that individual in the basic ordering system ... Usually the order is represented by an association of positions with numbers being viewed as 'coordinates' of the position, in which case numerical expressions or n-tuples of such ... appear as individual expressions" (Carnap 1958, pg. 161).

Coordinate languages provide means to represent the primary breakup of a spatial scene in primary objects of focus and secondary objects of reference in a formal language. The secondary objects of a scene, i.e., the ground, provide the basic ordering system. Coordinates represent the location of primary, i.e., figure objects, within the frame of reference provided by the ground.

In this paper we propose a coordinate language of regional individuals. Since we are in the domain of regions the numbers are replaced by names of regions and relations between regions. The basic ordering system is provided by the regional partition formed by the regions belonging to the ground.

The structure of a regional partition allows to distinguish the interior of a ground region and different parts of the boundary. For example, in Figure 1 we can distinguish the interior of the ground region `A` and the segment of boundary of `A` which is shared with the neighboring region `B`. An ‘interior boundary segment pair’ `(A,B)` refers to the interior of the ground region `A` and to a *single one dimensional* part of the boundary of `A` which is shared with the ground region `B`. If for a pair of regions more than one ‘interior boundary segment pair’ exist, they have to be uniquely named.

The base of the coordinate space of the coordinate language of a regional partition is formed by an arbitrary but fixed permutation of interior boundary segment pairs: $<(c_i, c_j), \ldots, (c_k, c_l)>$. Coordinate n-tuples corresponding to this coordinate base are defined as follows:

$$\mathrm{LOC(z)} \equiv_{def} (\mathrm{fg}_1, \ldots, \mathrm{fg}_n)$$
$$\text{if and only if z satisfies the formula}$$
$$(\mathrm{fg}_1((\mathrm{c}_i,\mathrm{c}_j),\mathrm{z}) \wedge \ldots \wedge \mathrm{fg}_n((\mathrm{c}_k,\mathrm{c}_l),\mathrm{z}))$$

where

- z is a figure region;
- LOC(z) represents the location of the figure region z within a partition formed by the regions $c_1, \ldots, c_n$;
- $(c_i, c_j), \ldots, (c_k, c_l)$ are the 'interior boundary segment pairs' of the regional partition formed by $c_1, \ldots, c_n$; $(i,j) \neq (k,l)$; The order of the $(c_i, c_j), \ldots, (c_k, c_l)$ in the coordinate base is arbitrary but fixed.
- $fg_i \in$ FG_rel;
- FG_rel is a finite set of 'figure ground relations' that hold between figure regions and parts of (ground) regions denoted by interior boundary segment pairs (Section 5.1, Table 1, Table 2);
- $(fg_1, \ldots, fg_n) \in$ FG_rel $\times \ldots \times$ FG_rel are coordinate n-tuples for a fixed basis of interior boundary segment pairs $<(c_i, c_j), \ldots, (c_k, c_l)>$ representing a regional partition.

Consider the universe of discourse of regional individuals in the plane. Consider the location of regional individuals within an ordering system, in particular within a finite partition of regions. Two individuals are equivalent with respect to their location within a fixed ordering frame of reference if and only if their location is represented by the same coordinate n-tuple:

$$\forall z_1, z_2 [z_1 \cong z_2 \leftrightarrow (\mathrm{LOC}(z_1) = \mathrm{LOC}(z_2))]$$

Sets of individuals with equivalent location partition the universe of discourse. The number of classes of equivalent location is finite if the number of different coordinates is finite. *Representing equivalence classes of individuals in a coordinate language of a finite coordinate space rather than individuals themselves allows to represent infinite domains by finite means.*

In the remaining sections we give the formal definitions for the concepts informally introduced above.

4 Customizing the RCC Theory

Our formalization is based on the theory of regions and connection - the RCC-8 theory (Randell, Cui et al. 1992). The RCC theory is based on the axiomatization of the primitive relation connectedness, C(x,y), which holds for pairs of regions. Using the primitive relation C 8 jointly exhaustive and pairwise disjoint relations are defined. Possible geometric interpretations of these relations were given in Figure 2. The axioms and definitions of the RCC theory and a discussion of their properties can be found for example in (Cohn, Bennett et al. 1997; Gotts, Goodday et al. 1996).

In order to make the RCC theory suitable to represent phenomena in geographic space (Egenhofer and Mark 1995), additional axioms have to be provided so as to make the theory more specific to the domain of phenomena in geographic space. The axioms have to be chosen in such a way that they focus on relevant aspects of the domain to be modeled and discard irrelevant aspects. Properties of axiom systems of spatial theories are discussed in (Lemon 1996).

From the point of view of representing phenomena in geographic space we concentrate on topologically well formed regions (no spikes, no single external points), which may be scattered and/or holed, and which are embedded in the Euclidean plane. Axioms which constrain models of the RCC theory to topologically well formed regions in the plane were given in (Bennett 1996). If we refer in the remainder of this paper to the RCC theory we mean the standard RCC-8 theory enriched by these axioms.

In the definitions below we explicitly use the predicates WCON, FEC, and FEC1 (Bennett 1996). WCON characterizes 'well connected', topologically simple regions (Figure 3). Complex regions consist of separated parts which are well connected regions.

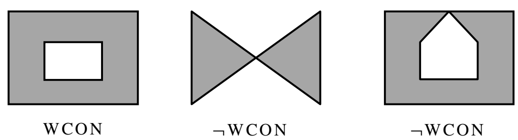

Figure 3: Well connected and not well connected regions (Gotts, Goodday et al. 1996)

FEC refers to well connected regions which are 'firmly externally connected via boundary segments'. FEC1 refers to well connected regions which are 'firmly externally connected via a single boundary segment' (Figure 4).

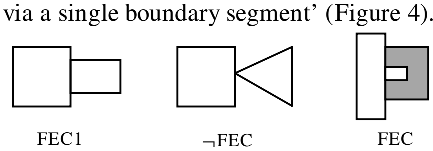

Figure 4: Externally connected regions

5 Figure Ground Relations

Figure ground relations are the domain of the 'coordinate values' which represent the location of a figure region within the ground represented by a regional partition. The base of the coordinate system is provided by an arbitrary but fixed permutation of the set of interior boundary segment pairs of the regional partition. Figure ground relations are relations between a figure region and ground regions denoted by an interior boundary segment pair. In the remainder of this paper we simplify and extend (Bittner 1996) .

5.1 The Definition of Figure Ground Relations

Figure ground relations are defined for triples of regions. The following properties are assumed to hold for triples of regions which belong to figure ground relations:

- Regions are topologically well formed and embedded in the Euclidean plane, topologically simple regions are well connected (section 4);
- Two regions (ground regions, denoted by (a,b)) are well connected, i.e., possibly holed, and firmly externally connected via a single boundary segment;
- The third region (figure region, denoted by *z*) may be arbitrarily scattered, i.e., $z= \mathrm{sum}(z_1,\ldots,z_n)$, $\mathrm{WCON}(z_i)$;

Consider Figure 1. Triples of regions for which figure ground relations hold are, for example: ((A,B),R),((A,B),S),((A,B),T),… . In general:

((ground_region1,ground_region2),figure_region)
and FEC1(ground_region1,ground_region2).

Ground_region1 (a) is the primary ground region. Ground_region2 (b) simply serves to name the boundary segment of a which is considered. Figure ground relations distinguish topologically different configurations of figure_region (z) and ground_region1 (a) concerning the way z occupies a and the boundary segment between a and b.

Figure ground relations are defined by the postulation of the existence and the denial of the existence of characteristic parts of a (Figure 5) which are either part (P) of z or disconnected (DC) to z:

$$+\mathrm{DC} \equiv_{\mathrm{def}} \exists x[\mathrm{DC}(x,z)];\; x \in \{r1,r2\}$$
$$-\mathrm{DC} \equiv_{\mathrm{def}} \neg\exists x[\mathrm{DC}(x,z)];\; x \in \{r1,r2\}$$
$$+\mathrm{P} \equiv_{\mathrm{def}} \exists x[\mathrm{P}(x,z)];\; x \in \{r1,r2\}$$
$$-\mathrm{P} \equiv_{\mathrm{def}} \neg\exists x[\mathrm{P}(x,z)];\; x \in \{r1,r2\}$$

The identifiers r1, r2 denote arbitrary regions which are topologically equivalent to the regions r1,r2 in the configuration in Figure 5.

+DC postulates the existence of a region of type r1 or r2 which is disconnected to the figure region z. −DC denies the existence of such a region. +P postulates the existence of a region of type r1 or r2 which is part of the figure region z. −P denies the existence of such a region.

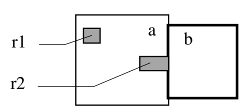

Figure 5: Characteristic parts of an 'interior boundary segment pair'

In Table 1 the figure ground relations are defined by applying the primitives +DC,−DC,+P,−P to regions of type r1 and r2 as shown in Figure 5. Each row of Table 1 is an abbreviation for a statement of the following form (row 3 as an example):

The relation FG_PBO (partial boundary occupation) holds for a triple of regions a,b,z if and only if

- the relation FEC1(a,b) holds, and

- there exists a region of type `r1`, which is a proper part of `a`, and which is part of the region `z`, and
- there does exist a region of type `r2`, which is part of `a` and which is externally connected to `b`, and which is part of `z`, and
- there does exist a region of type `r2`, which is part of `a` and which is externally connected to `b`, and which is disconnected from `z`.

English term	Predicate	r1	r2
no occupation	FG_NO	−P	−P
no boundary occupation	FG_NBO	+P	−P
partial boundary occupation	FG_PBO	+P	+P,+DC
full boundary occupation	FG_FBO	+DC	−DC
full occupation	FG_FO	−DC	−DC

Table 1: The definition of figure ground relations

Each row in Table 1 can be expanded to a logical formula of the RCC theory in the following manner (row 3 as an example):

$$
\begin{aligned}
\mathrm{FG\_PBO}((a,b),z) \equiv_{def}\ & \mathrm{FEC1}(a,b)\ \wedge \\
& \exists r1[\mathrm{PP}(r1,a) \wedge \mathbf{P}(r1,z)]\ \wedge \\
& \exists r2[\mathrm{TPP}(r2,a) \wedge \mathrm{EC}(r2,b) \wedge \mathbf{P}(r2,z)]\ \wedge \\
& \exists r2'[\mathrm{TPP}(r2',a) \wedge \mathrm{EC}(r2',b) \wedge \mathbf{DC}(r2',z)]
\end{aligned}
$$

In Table 2 possible geometric interpretations for the figure ground relations are given:

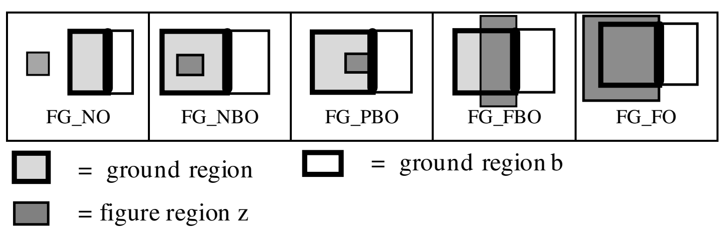

Table 2: Possible geometric interpretations of the figure ground relations

5.2 Properties of Figure Ground Relations

The set of figure ground relations has the property of being jointly exhaustive and pairwise disjoint, i.e. JEPD. JEPD means that for arbitrary triples of regions for which figure ground relations are defined *one and only one* figure ground relation holds.

Consider Table 1. Each figure ground relation is defined by a *unique* pattern (no row occurs twice in Table 1) of postulation of existence or denial of the existence of characteristic regions (Figure 5) with particular relations of connectedness to a particular figure region. Consequently, the defined relations must be pairwise disjoint.

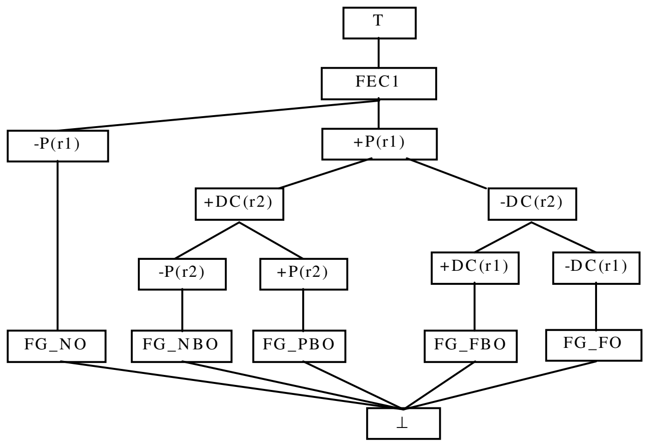

Figure 6: The lattice structure of the definitions of the figure ground relations

Consider Figure 6. The lattice structure formed by the definitions of the figure ground relations is shown. Every node in the lattice below `FEC1` corresponds to the postulation or denial of the existence `(+,-)` of a characteristic region of type `(r1)`, `(r2)` (Figure 5) with particular relations of connectedness (`DC` or `P`) to a figure region. This *structure* of the definitions ensures that for *arbitrary* triples of regions for which figure ground relations are defined *one and only one* figure ground relation holds.

6 Figure Ground Location

6.1 Configurations of Regions Forming a Partition of Space

In this section a formal definition of regional partitions of space which are formed by a finite number of well connected (ground) regions is given. `CONF` ensures that the regions belonging to a configuration of regions forming a regional partition of the plane are well connected and that if they are distinct and connected then they are externally connected. The sum of all configuration regions sums up the whole universe.

$$\mathrm{CONF}(x_1,\ldots,x_n) \equiv_{def}$$
$$\mathrm{WCON}(x_i) \wedge (\forall y[\mathrm{C}(y,\mathrm{sum}(x_1,\ldots,x_n))]) \wedge$$
$$(\mathrm{C}(x_i,x_j) \rightarrow \mathrm{EC}(x_i,x_j))\ i,j,n \in N,\ i \neq j\ ,$$
$$1 \leq i \leq n,\ 1 \leq j \leq n,\ n > 1.$$

In the remainder we assume that we refer to an *arbitrary but fixed* partition of regions satisfying the CONF predicate. Otherwise every statement about location would need additional parameters specifying the particular partition it is referring to.

6.2 Location within Regional Partitions

Definitions

Consider the theory, FG−LOC, of a language of figure ground relations enriched by a finite set of constant symbols $c_1, \ldots, c_n$ denoting the elements of a regional partition $\underline{c}_1, \ldots, \underline{c}_n$. This theory assumes the RCC theory (RCC-8 constrained to topologically well formed regions in the plane) and enriched by the definitions of the sections 5.1 and 6.1. Assume an axiom postulating a particular partition belonging to FG_LOC:

$$\mathrm{CONF}(\ c_1, \ldots, c_n\ ),\ n \in N,\ n>1\ .$$

For simplification we rule out partitions in which two regions are externally connected via more than one boundary segment by adding the following axiom to FG_LOC:

$$\mathrm{FEC}(c_i, c_j) \rightarrow \mathrm{FEC1}(c_i, c_j);\ 1 \leq i \leq n,\ 1 \leq j \leq n,\ i \neq j,\ n \in N,\ n>1.$$

This has the advantage that every boundary segment can be uniquely named by a pair of neighboring ground regions. Otherwise we had to distinguish separated boundary segments.

Every theory of this kind contains a finite formula of the following form:

$$\forall z [\ \bigwedge\ (\ \mathrm{FEC1}(c_i, c_j) \rightarrow (\mathrm{FG\_NO}((c_i, c_j), z) \vee \ldots \vee \mathrm{FG\_FO}((c_i, c_j), z)))];$$
$$1 \leq i \leq n,\ 1 \leq j \leq n,\ i \neq j,\ n \in N,\ n>1.$$

Within a particular (consistent) theory for every regional individual one and only one ground formula (all variables are replaced by individual constants) of this form can be found (ignoring the order in which sub formulas occur). These are consequences of the JEPD property of the figure ground relations (Section 5.2) and the partition structure axiomatized above.

To get ground formulae of minimal length the following simplifications are made:

- Only conjunctions referring to pairs of ground regions for which FEC1 holds are mentioned explicitly.
- The FEC1 property is not mentioned explicitly.
- From every disjunction of figure ground relations only the one relation which actually holds is mentioned explicitly.

Under these preconditions for arbitrary regional individuals a valid conjunction of figure ground relations exists. Minimal formulae of conjunctions of figure ground relations have the following form:

$$\mathrm{LOC'(z)} \equiv_{def} \mathrm{fg_1((c_i,c_j),z)} \wedge \ldots \wedge \mathrm{fg_n((c_k,c_l),z)}$$
$$\mathrm{fg_1,\ldots,fg_n} \in \{\mathrm{FG\_NO,FG\_NBO,FG\_PBO,FG\_FBO,FG\_FO}\}$$
$$1 \leq i,j,k,l \leq n,\ i \neq j,\ k \neq l\ n \in N,\ n>1.$$

For every region one and only one valid conjunction of figure ground relations exists. Different regions can satisfy the same formula.

Coordinate Representation of Figure Ground Location

Let $\mathrm{base} = \langle (c_i,c_j),\ldots,(c_k,c_l) \rangle$ be an arbitrary but fixed permutation of interior boundary segment pairs of a particular regional partition. The n-tuple $\mathrm{LOC_{base}(z)=(fg_1,\ldots,fg_n)}$ is a coordinate representing the location of a figure region z with respect to base if and only if z satisfies FG-LOC enriched by the formula $\mathrm{LOC'(z)}$:

$$\mathrm{LOC_{base}(z)} =_{def} (\mathrm{fg_1,\ldots,fg_n}) \leftrightarrow$$
$$((\{\underline{c}_1,\ldots,\underline{c}_n,\underline{z}\},\underline{C}) \models (\mathrm{FG\text{-}LOC} \cup \{\mathrm{LOC'(z)}\}))$$

where:

- $\underline{c}_1,\ldots,\underline{c}_n$ are ground regions denoted by the constants $c_1,\ldots,c_n$
- $\underline{z}$ is the figure region which is assigned to the variable z and which location is represented
- $\underline{C}$ is the relation of connectedness between regions, the interpretation of the language primitive C
- $\mathrm{FG\text{-}LOC} \cup \{\mathrm{LOC'(z)}\}$ is a set of logical formula containing the definitions and axioms discussed in the previous sections enriched by the formula $\mathrm{LOC'(z)}$.
- $\models$ states that the set of formulas $\mathrm{FG\text{-}LOC} \cup \{\mathrm{LOC'(z)}\}$ is satisfied in the 'model of connectedness' $(\{\underline{c}_1,\ldots,\underline{c}_n,\underline{z}\},\underline{C})$ of the regional partition $\{\underline{c}_1,\ldots,\underline{c}_n\}$ and the figure region $\underline{z}$.

Equivalence of Location

Regions that are represented by the same coordinates, i.e., satisfy the same conjunction of figure ground relations, are considered to be equivalent with respect to their location in the corresponding regional partition:

$$\forall x,y[x \cong_{LOC} y \equiv_{def} \mathrm{LOC(x)=LOC(y)}].$$

$\cong_{LOC}$ is an equivalence relation, i.e., reflexive, symmetric, and transitive.

Equivalence relations partition the universe of discourse of the individuals over which they are defined. The *infinite* universe of topologically well formed regions which are embedded in the plane is decomposed into a *finite* number of *classes of regions* with *equivalent location* within a regional partition.

6.3 Examples

Location in a Regional Partition

Consider a language of figure ground relations whose signature is enriched by the constant symbols `A,…,E`. We add the axiom

```
CONF(A,B,C,D,E,compl(sum(A...E)))
```

to the theory.

Consider Figure 1. The location of the figure regions `R,S,T` within the regional partition `GR={A,B,C,D,E,compl(sum(A...E)}` is represented by coordinate n-tuples in a coordinate space formed by a base of interior boundary segment pairs (Example 3).

```
LOC(R)= LOC(S) = (PBO,PBO,PBO,PBO,NBO,NO,PBO,PBO,
        NO,NO,NO,NO,NBO,NO,NBO,NO,NO,NO,NO,NO,NBO,
        NO)
LOC(T)= (NO,NO,NO,NO,NO,NBO,NO,NO,NO,NO,NBO,NO,
        NO,NO,NO,NO,NBO,NO,NO,NO,NO,NO)
Base = <(A,B),(B,A),(A,E),(E,A),(B,C),(C,B),
        (B,E),(E,B),(E,D),(D,E),(C,D),(D,C),
        (A,Ext),(Ext,A),(B,Ext),(Ext,B),(C,Ext),
        (Ext,C), (D,Ext),(Ext,D),(E,Ext), (Ext,E)>
Ext = compl(sum(A...E)))
```

Example 3**:** Coordinate representation of location of the figure regions within the ground (Figure 1). The prefix FG_ of the figure ground relations is omitted.

The regions `R` and `S` cannot be distinguished in terms of the language of the theory of `GR`, `Th(GR)`. They are equivalent within `Th(GR)`: $R \cong_{\mathrm{LOC\ in\ TH(GR)}} S$ They can be distinguished within the RCC theory: $\mathrm{PO(R,S)} \rightarrow \neg\mathrm{EQ(R,S)}$.

Pairs of Figure Ground Relations

Consider location in a coordinate space formed an arbitrary finite partition of regions. Consider the pair (c_i, c_j) of `FEC1` ground regions. Consider the corresponding interior boundary segment pairs in the coordinate base, e.g., $<\ldots, (c_i, c_j), (c_j, c_i), \ldots>$. Consider the set of fragments of coordinate n-tuples $<\ldots, fg_m, fg_n, \ldots>$. We only consider fragments of coordinate n-tuples which refer to figure regions `z` which overlap the sum of (c_i, c_j), are connected to c_i, and do not fully occupy c_j.

In Table 3 the expanded definitions of coordinate fragments based on Table 1 are shown. The identifiers `r1` and `r2` refer to characteristic regions in c_i (Figure 5). The identifiers $\underline{\mathtt{r1}}$ and $\underline{\mathtt{r2}}$ refer to characteristic regions in c_j. In Table 4 possible geometric interpretations of the coordinate fragments in Table 3 are given.

Using the definitions in Table 3 it is impossible to distinguish between the two configurations in Figure 7.

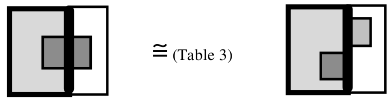

Figure 7: Configurations of <…,PBO,PBO,…>

Disconnected parts of scattered figure regions can only be distinguished if they are separated by boundaries of ground regions. The representation of the topological structure of figure regions may be undetermined due to the fact that only *topologically significant relations between figure and ground and not between parts of the figure* are considered. The 'finer' the structure of the ground the better the topological structure of the figure regions can be represented.

	r1	r2	$\underline{\text{r1}}$	$\underline{\text{r2}}$
<…,NBO,NO,…>	+P	−P	−P	−P
<…,PBO,NO,…>	+P	+P,+DC	−P	−P
<…,PBO,PBO,…>	+P	+P,+DC	+P	+P,+DC
<…,NO,PBO,…>	−P	−P	+P	+P,+DC
<…,NBO,PBO,…>	+P	−P	+P	+P,+DC
<…,FO,FBO,…>	−DC	−DC	+DC	−DC
<…,FO,NO,…>	−DC	−DC	−P	−P
<…,FO,PBO,…>	−DC	−DC	+P	+P,+DC
<…,FBO,NO,…>	+DC	−DC	−P	−P
<…,FBO,FBO,…>	+DC	−DC	+DC	−DC
<…,FBO,PBO,…>	+DC	−DC	+P	+P,+DC
<…,PBO,FBO,…>	+DC	+P,+DC	+DC	−DC
<…,NO,FBO,…>	−P	−P	+DC	−DC
<…,NBO,FBO,…>	+P	−P	+DC	−DC

Table 3: Expanded definitions of figure ground coordinate fragments

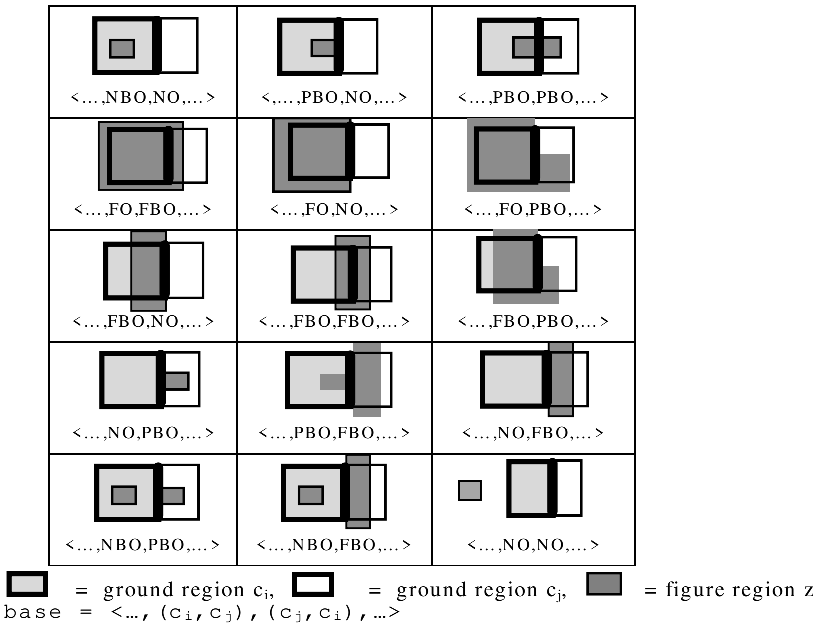

Table 4: Possible geometric interpretations of the figure ground coordinate fragments in Table 3.

7 Conclusions

In this paper we proposed a coordinate language of regional individuals. We showed that this language provides means to represent the primary breakup of a spatial scene in primary objects of focus and secondary objects of reference, arising in natural language, in a formal language. At the formal level an infinite theory of individuals is replaced by a finite theory of location of individuals.

7.1 A Finite Theory of Location of Figures within the Ground

We constructed a formal theory of location of figures within the ground, i.e., of location of regional figures within a frame of reference provided by a regional partition of the plane, the ground. This theory is based on the RCC theory. The construction was done in five steps:

1. The models of the RCC theory were constrained to topologically well formed regions that are embedded in the Euclidean plane.
2. The language of the RCC theory was enriched by predicate symbols denoting figure ground relations which hold for triples of regions. Figure ground relations were defined in terms of connectedness based on the definitions of the RCC theory.

3. This language was further enriched by constant symbols denoting regions which form a regional partition of the plane. An axiom was added postulating the particular neighborhood structure of a regional partition.
4. The notion of *location of a region* in an arbitrary but fixed regional *partition of regions* was defined.
5. A coordinate language was constructed by considering n-tuples of figure ground relations as *coordinates* denoting locations within a *coordinate space* corresponding to a regional partition.

The theory of location of regions in a finite partition of the plane is finite. There exist only a finite number of locations that can be distinguished in terms of the language of the theory. The infinite set of regions forming the universe of discourse is partitioned into a finite set of classes equivalent location. These equivalence classes are the individuals which are represented in terms of location in the coordinate language of figure ground location.

7.2 Relevance for Reasoning in Geographic Space

The representation of qualitative location of equivalence classes of individuals rather than individuals themselves is relevant for the formalization of large scale (geographic) space due to the following reasons:

- Regional partitions are often results of conceptualization of non-bounded phenomena in geographic space. Location of bounded regional phenomena within a regional partition can be considered as a formal representation of relations between bounded and non-bounded phenomena.
- The character of geographic space being large scale space implies that a potentially large amount of individuals are involved in the representation and reasoning process. Coordinate languages of figure ground location provide means to structure a large amount of individuals and to handle finite (or small) sets of equivalence classes of individuals rather than infinite (or large) sets of individuals.
- Coordinate languages of figure ground locations provide means to encode knowledge about the structure of geographic space into the language, in particular into the base of the coordinate system.
- Qualitative coordinate languages allow to abstract from measurement of the location of the boundary phenomena whose boundaries cannot be measured.

7.3 Some Open Problems

There are a number of open questions that need to be considered:

1. Which properties of the original domain of individuals are preserved when moving to the domain of equivalence classes of individuals. For example: Is $\cong_{LOC}$ compatible with the operations sum, prod, compl, e.g. $[\texttt{prod(x,y)}]_{LOC} = \texttt{prod}_{LOC}([\texttt{x}]_{LOC}, [\texttt{y}]_{LOC})$?
2. What information about relations between individuals can be derived from knowledge about location ?

3. How can additional knowledge about the structure of the regional partition, e.g., regularity of structure, properties of order and metric, be used in the reasoning process ?

8 Acknowledgments

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