A Qualitative Model of Geographic Space[1]

Thomas Bittner
Department of Geoinformation
Technical University of Vienna
Gusshausstrasse 27-29
A - 1040 Wien
bittner@geoinfo.tuwien.ac.at

Abstract

In this paper a qualitative model of geographic space is proposed. The model consists of two types of primitives, covers and regions, and relations between them. A cover is a set of regions that form a partition of space. Spatial entities are represented by regions. The primitives correspond either to the field view or to the entity-oriented view of geographic space. The primitives and relations between them are formalized in terms of topology. The relation of a region with respect to a cover is represented formally in terms of topological relations between the region and elements of the cover. This set of relations can be considered as a qualitative representation of a region within a cover. This representation qualitatively specifies the location of a spatial entity, abstracted by a region, within its environment, abstracted by a cover.

Keywords: Qualitative Spatial Reasoning

1. Introduction

In GIS natural, physical, biological, or social phenomena are considered regarding their relation to their spatial environment. Those phenomena for example are temperature, soil type, soil wetness, and slope of terrain. Laurini & Thompson characterize the treatment of such phenomena in GIS in the following way (Laurini and Thompson 1994):

Natural, physical, biological phenomena are often continuously variable and continuously distributed over space. Social phenomena may be considered as spatially continuous at a particular scale. These phenomena are usually treated by the field view of space. Continuous distribution of phenomena over space is the principal element of this view of space.

> ”...areal units are delimited somehow so that boundaries can be drawn for polygonal entities, producing land use zones, and the like. The attribute information is carried by the area elements, and the boundaries reflect varying assumptions of change between adjacent units. A sharp break or discontinuity is generally assumed, without any possibility of providing a mathematical statement for the differences, and varying assumptions are made about variation within the units” (Laurini and Thompson 1994) pg. 89.

The need for a sharp break or discontinuity at the boundaries is due to the representation of the boundaries by means of geometric objects, that are formalized analytically in terms of Euclidean geometry. Analytical Euclidean geometry is an abstract formal system that is based on measurement

[1] Bittner, T. (1996). A Qualitative Model of Geographic Space. SDH’96, Delft, Netherlands (August 12-16, 1996), IGU.

along coordinate axes. Abstract spaces are no real spaces. They only exist in our minds. Sometimes there exists a correspondence between abstract and real space, for example between Euclidean space and table-top spaces (Nunes 1991). There does not exist a correspondence between the natural phenomena in geographic space (Mark and Frank 1995) and abstract geometric primitives in Euclidean space (Couclelis 1992). This becomes obvious in the contradiction between the assumption of continuity of distribution over space and the 'need' for discontinuities or sharp changes at the boundaries that have to be describable in terms of geometric primitives.
For the formalization of natural phenomena in geographic space we need theories of spaces that do not constrain their models to sharp bounded geometric objects. In this approach topological spaces are used for the formalization. Topological spaces allow us to represent boundaries without forcing them into geometric primitives by declaring metric positions.
Similar problems occur if we apply the entity-oriented view to natural phenomena in geographic space. In this view space is conceptualized as a set of entities that correspond to phenomena described by a pattern of attributes. The spatial extend of an entity is considered as one of its attributes.
If we represent for example an area of pollution in terms of geometric primitives using analytical formalization of Euclidean geometry we are required to draw sharp boundaries where in nature are continuous changes and/or the precise metric position of the boundary is unknown. A similar problem is to decide where a valley ends and a mountain begins (Couclelis 1992).
In nature there are neither entities nor fields. The distinction between different views of space is due to different conceptualizations of phenomena in geographic space made by men (Couclelis 1992). How geographic space is conceptualized depends for example on the geographic scale of the space under consideration (Zubin 1989), the purpose of the conceptualization (Couclelis 1992), and the nature of phenomena (examples above).
GIS can be seen as the implementation of a formal theory about geographic space (Mark and Frank 1995). In this theory one should take the different conceptualizations of geographic space into account. Since the phenomena in the geographic world are interrelated independently of their conceptualization by men, models based on their different conceptualizations should be interrelated too. Therefore the representations of both views should be related to each other and formalized in a unified theory.
In this paper I propose a representation scheme that incorporates both views of geographic space. The formalisation is based on topology and results in a qualitative representation (Freksa 1991). Since this representation does not rely on coordinates and geometric primitives, there is no need to measure and to represent unknown metric boundary positions.
I concentrate on problems where the application of the field view results in a regional subdivision of space. Thus the application of the field view results in a set of regions that are either disjoint or meet each other, which is called a *cover.* A *cover* corresponds to the concrete spatial structure of a particular environment, extracted by application of the field view on particular aspects of this environment. The spatial extend of entities is abstracted by non-overlapping regions. Relations between a *cover* and a region are defined qualitatively in terms of topology. This relations topologically specify the location of a region with respect to a structured set of regions - the *cover* .
This approach differs from other formalizations of geographic space in the following points:

- This approach is based on topology. Thus it is qualitative and no (coordinate) measurement is involved
- The cover structure that corresponds to a field view of a particular environment is used as a reference system for the representation of knowledge about spatial entities within this environment. Thus spatial entities are represented formally within a concrete space rather than within an abstract space.
- Qualitative spatial reasoning usually takes abstract relational structure of the spatial domain into consideration (i.e. topological relations, conceptual neighborhood) (Egenhofer 1991; Frank 1991; Freksa 1991; and Freksa 1992; Hernandez 1994) rather than the concrete structure of a particular environment, for example, in terms of a *cover* structure (Bittner 1995).

The paper is organized as follows: In the second section the conceptual model is introduced. In the third section basic point set topological notions and binary topological relations between regions are explained. Covers, regions and relations between them are formalized in the fourth section. A summary and examples are given in the last sections.

2. The Conceptual Model

The conceptual model is intended to cover some aspects of geographic space and its conceptualization that are relevant for GIS. In this model geographic space is modeled as a set of two types of primitives. Instances of the primitives of the model are created by the conceptualization of phenomena in geographic space. Corresponding to the two views of geographic space the two basic types of primitives in this model are (Figure 4):

- Entity-regions

  The spatial extend of a geographic entity is represented by one region or more regions that may have holes but do not overlap. The representation of geographic entities by regions is due to the assumption that all objects in real space have an extension in three dimensions which projections on Earth's surface result in regions.
- Cover

  The field view of a particular attribute varying continuously over space is represented by a set of regions that either meet each other or are disjoint and form a partition. This set of regions is called a *cover*. Within each region the attribute values belong to the same equivalence class in the attribute domain. The regional partition of space corresponds to the equivalence classes in the attribute domain.

Instances of these primitives are objects in that sense that they are characterized by the prototypical object qualities of discrete identity, relative permanence of structure and attribute, and the potential of being manipulated. The important point is that not the soil-structure of a particular environment is the object, but the cover that represents a particular view of the soil structure is the object.
In this model geographic space is considered as being relative (Nunes 1991). Thus it consists of primitives and relations between them.
The geographic space is modeled by :

- a set of covers, each cover corresponding to the field view of one attribute varying continuously with respect to Earth' surface;
- a set of entity-regions, corresponding to spatial entities;
- relations between covers;
- relations between a cover and entity-regions;
- and relations between entity-regions.

Geographic phenomena that can be represented by a cover are for example soil classifications, political boundaries, ownership... . Buildings, parcels, pollution areas, lakes, ... can be represented as entity-regions. Relations between covers correspond to results of overlay operations in current coordinate based GIS. Relations between a cover and an entity-region are for example containment of an entity in a region of the cover ('Vienna lies in Austria' where Austria belongs to the cover 'Countries in Europe'). These relations specify location. Relations between entity-regions are for example the binary topological relationships (e.g. disjoint, meet,...). In Euclidean geometry based GIS covers are represented by layers of polygons, where regions are represented by polygons which consist of boundary-segments that are shared by neighboring polygons.

3. Point Set Topology and Binary Topological Relationships

The primitives of the model and the relations between them are defined in terms of point set topology. Their formal definition is based on the point set topological notions of interior and boundary (Spanier 1966). The definitions are based on the notion of open sets. The interior of a point set Y, denoted by $Y°$, is the union of all open sets that are contained in Y. The *closure* of Y, denoted by $\overline{Y}$, is defined by the intersection of all closed sets that contain Y. The *boundary* of Y, denoted by δY , is the intersection of the closure of Y and the closure of the complement of Y. A *separation* of Y , $Y \subseteq X$, is a pair A, B of subsets of X satisfying the condition $A \neq \emptyset$ and $B \neq \emptyset$; $A \cup B = Y$; and $\overline{A} \cap B = \emptyset$ and $A \cap \overline{B} = \emptyset$. Y is said to be disconnected, otherwise Y is said to be connected. A *region* is a non-empty connected set in $\mathbf{R}^2$ (Egenhofer and Herring 1990).
Topological Relationships between regions in $\mathbf{R}^2$ were formalized by Egenhofer and Herring (Egenhofer and Herring 1990). The topological invariance of the intersection of the boundary and the

interior of two regions are used to define topological relationships between them. Egenhofer and Herring show that from the 16 combinatorically possible intersection pattern only eight can exist between two regions with codimension 0. These are geometrically interpreted and identified with the topological distinguishable relationships between two regions without holes in $\mathbf{R}^2$. The eight topological relations are : disjoint, meet, equal, inside, covered by, contains, covers and overlap.
In this approach a similar technique is applied to define topological relationships between a cover and an entity-region.

4. Relations between Covers and Regions

4.1. The Cover-structure

4.1.1. The Definition of Cover-structures

Let *cover* be a set of regions in $\mathbf{R}^2$ forming a finite regional partition**.** $\mathbf{R}^2$ is covered by the union of all cover-regions and the complement of this union. A region A is a non-empty connected set of points. It consists of two disjoint subsets: the interior, denoted by A° and the boundary, denoted by δA, such that $A=A°\cup\delta A$ and $A°\cap\delta A=\emptyset$. All regions that belong to a cover are either disjoint or meet each other, where disjoint means that they do not have points in common and meet means that they share common boundary points.
The boundary δA of each region consists of one or more than one *boundary segments*, denoted $\delta A_\#$. Each boundary segment is a connected subset of the boundary of a region. A boundary segment δA_B is the non-empty intersection of the boundaries of two adjacent regions A and B. There are boundary segments which belong to the outer boundary of the *cover*, denoted $\delta A_\emptyset$. They are shared with the complement of the *cover*. $\delta A_\#$ denotes an arbitrary boundary segment $\delta A_\# \subseteq \delta A$ (either δA_B or $\delta A_\emptyset$).
If there exist more than one separated boundary segments between two neighboring regions or a region and the complement of the *cover* , then they are uniquely numbered.
A *cover-structure* is the representation of a *cover*. It is the set of all pairs of boundary segments and the corresponding interior of a region that belong to the cover. Thus for each boundary segment $\delta A_B=\delta B_A$ two pairs $(A°,\delta A_B)$ and $(B°,\delta B_A)$ exist within the *cover-structure*.
Definition 1 (*cover-structure*) :

Let *cover* be a finite regional partition, $A\in$ *cover*, and $(A°,\delta A_\#)$ a pair, called interior-boundarysegment-pair , where

- A° denotes the interior of the region A,
- $\delta A_\#$ denotes a boundary segment of the region A.

A *cover-structure* is the set of all pairs $(A°,\delta A_\#)$ definable for regions of a cover:

$$\textit{cover-structure} =_{\text{Def}} \{(A°,\delta A_\#) \mid A \in \textit{cover} \ \& \ \delta A_\# \subseteq \delta A\}.$$

4.1.2. Cover-structures vs. Cell Complexes

Cover-structures and cell-complexes (Alexandroff 1961; Frank and Kuhn 1986) are both formalizations of regional partitions of space. Cell-complexes have a simple and regular structure. Due to the regular structure of cell-complexes in general no one to one correspondence between the cells and units of the conceptual model can be established. Thus meaningful units have to be approximated by sets of cells. The elements of the cover-structures directly correspond to the smallest meaningful units of the conceptual model : boundary segments and interior of regions that constitute a cover.
Equivalent to the point set topological definition the *cover-structure* could be defined in terms cell-complexes based on algebraic topology.

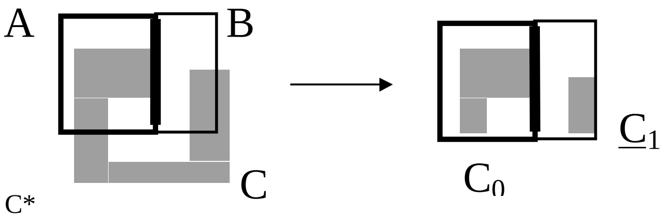
4.2. Relations Between a Cover and a Region

Entity regions in this formalization are single regions without holes. In Definition 1 pairs $(A°,\delta A_\#)$ are used as primitives of the *cover-structure*. The elements A° and $\delta A_\#$ refer to subsets of a region A. A belongs to the *cover*. The region C represents a spatial entity which consists of the union of the two disjoint subsets: interior and boundary, $C=C°\cup\delta C$. The topological relations between a *cover* and an entity-region are defined in terms of topological relations between sets denoted by pairs $(A°,\delta A_\#)$ of the *cover-structure* and the entity-region C. The topological relation between a pair $(A°,\delta A_\#)\in$ *cover-*

structure and an entity-region C is defined in terms of the content of the intersection of the following subsets:

- emptiness / non-emptiness of the intersection of the interior of A and the interior of C : $A°\cap C°$,
- emptiness / non-emptiness of the intersection of the boundary segment $\delta A_{\#}$ of A and the interior of C : $\delta A_{\#}\cap C°$,
- emptiness / non-emptiness of the intersection of the interior of A and the boundary of C : $A°\cap\delta C$,
- emptiness / non-emptiness of the intersection of the boundary segment $\delta A_{\#}$ of A and the boundary of C : $\delta A_{\#}\cap\delta C$.

Since we do not consider the whole boundary δA of A, we cannot handle C as a whole. If we intersect $(A°,\delta A_{\#})$ and C then a number of separated subsets of C can be created (Figure 1). We define a *connected intersection* $\underline{C}$ of C as a connected subset of C which is created by the intersection of C with the interiors and the shared boundary segment of two neighboring *cover*-regions. The set of all connected intersections $\underline{C}$ of C that can be created by the intersection of C with neighboring *cover*-regions or with a *cover*-region and the adjacent complement of the *cover* is called *connected intersection set* of C, denoted by C *. Each connected intersection $\underline{C}_i \in C^*$ consists of the union of two disjoint subsets $\underline{C}_i°$ and $\delta\underline{C}_i$, where $\underline{C}_i°\subseteq C°$, and $\delta\underline{C}_i \subseteq \delta C$.



$\underline{C}_0, \underline{C}_1 \in C^*$

Figure 1 : Connected intersections $\underline{C}_0$ and $\underline{C}_1$ of a region C that are created by the intersection of C with the interior of two neighboring regions A and B and their common boundary segment $\delta A_B=\delta B_A$.

We define the topological relations between a cover and an entity-region as the set of topological relations between the elements of the connected intersection set of the entity-region C, $\underline{C}_i\in C^*$, and the elements $(A°,\delta A_{\#})$ of the *cover-structure*. Before we can define this set of topological relations formally we need to define the topological relations between a single connected intersection and a single interior-boundarysegment-pair.

4.3. The Definition of CR-Relations

The topological relations between a pair $(A°,\delta A_{\#})\in$ *cover-structure* and a connected intersection $\underline{C}_i\in C^*$ of an entity-region C are defined in terms of the content of the intersection of the four sets $A°$, $\delta A_{\#}$, $\underline{C}_i°$, and $\delta\underline{C}_i$. There are 16 combinatorically possible intersection pattern (Table 1). These pattern are used for the definition of the topological relationships:

number	$\delta A_{\#}\cap\delta C_i$	$\delta A_{\#}\cap C_i°$	$A°\cap\delta C_i$	$A°\cap C_i°$	classification
$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	CR-empty
1	$\neg\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	R3
2	$\varnothing$	$\neg\varnothing$	$\varnothing$	$\varnothing$	L0
3	$\neg\varnothing$	$\neg\varnothing$	$\varnothing$	$\varnothing$	L1
4	$\varnothing$	$\varnothing$	$\neg\varnothing$	$\varnothing$	L2
5	$\neg\varnothing$	$\varnothing$	$\neg\varnothing$	$\varnothing$	L3
6	$\varnothing$	$\neg\varnothing$	$\neg\varnothing$	$\varnothing$	L4
7	$\neg\varnothing$	$\neg\varnothing$	$\neg\varnothing$	$\varnothing$	L5
8	$\varnothing$	$\varnothing$	$\varnothing$	$\neg\varnothing$	L6
9	$\neg\varnothing$	$\varnothing$	$\varnothing$	$\neg\varnothing$	E0
10	$\varnothing$	$\neg\varnothing$	$\varnothing$	$\neg\varnothing$	E1
11	$\neg\varnothing$	$\neg\varnothing$	$\varnothing$	$\neg\varnothing$	E2
12	$\varnothing$	$\varnothing$	$\neg\varnothing$	$\neg\varnothing$	R0

number	$\delta A_{\#} \cap \delta C_i$	$\delta A_{\#} \cap C_i°$	$A° \cap \delta C_i$	$A° \cap C_i°$	classification
13	$\neg\emptyset$	$\emptyset$	$\neg\emptyset$	$\neg\emptyset$	R1
14	$\emptyset$	$\neg\emptyset$	$\neg\emptyset$	$\neg\emptyset$	E3
15	$\neg\emptyset$	$\neg\emptyset$	$\neg\emptyset$	$\neg\emptyset$	R2

Table 2: The Intersection Table.

The intersection pattern can be divided into four classes. The pattern in class one, {L0...L6}, do not exist for non-holed entity-regions in 2D (Bittner 1996)(Lemma1,Lemma2). The second class, {E0...E3), contains the pattern that imply that the boundary segment $\delta A_{\#}$ under consideration is a proper subset of the connected intersection $\underline{C}_i$. The pattern E1 and E3 imply that the boundary segment $\delta A_{\#}$ is a proper subset of the interior of the connected intersection $\underline{C}_i$. The third class of pattern {R0,R1,R2,R3} imply a non-empty intersection between $(A°,\delta A_{\#}) \in$ *cover-structure* and $\underline{C}_i \in C^*$. The boundary segment $\delta A_{\#}$ is not contained within the connected intersection $\underline{C}_i$. Class four contains the pattern that corresponds to the empty intersection between $(A°,\delta A_{\#})$ and $\underline{C}_i$ and is called CR-empty. In Figure 2 possible geometrical realizations of the relations {R0..R3,E0..E3} are shown.

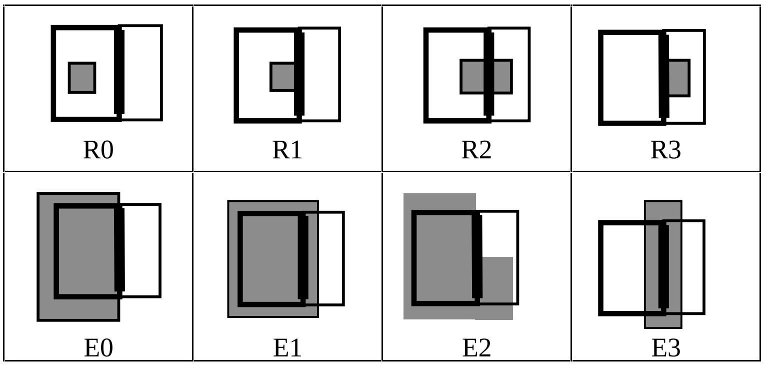


Figure 2: Geometric Interpretation of the CR- relations. The relations hold for the intersection of the dark entity-region C and the interior of the two cover-regions that meet each other (A denotes the left and # denotes the right region) and their common boundary $(\delta A_{\#})$.

Now we can express the topological relations between a pair $(A°,\delta A_{\#}) \in$ *cover-structure* and a connected intersection $\underline{C}_i \in C^*$ in terms of CR- relations:

Definition 2 ((C)over element - entity (R)egion - relations) :

The topological relationships, between a pair $(A°,\delta A_{\#})$, that denotes the interior and one boundary segment of a cover-region A, and a connected intersection $\underline{C}_i \in C^*$ of an entity-region C, that correspond to the realizable pattern of the content of the intersection of the sets $A°$, $\delta A_{\#}$, $\underline{C}_i°$, and $\delta\underline{C}_i$ (Table 1), are called *CR-relations*:

$$\text{CR-relations} =_{Def} \{ R0, R1, R2, R3, E0, E1, E2, E3, \text{CR-empty} \}.$$

A CR-relation is a binary relation that holds between a pair $(A°,\delta A_{\#})$ and a connected intersection $\underline{C}_i \in C^*$:

$$<(A°,\delta A_{\#}), \underline{C}_i > \in \text{CR-relations} =_{Def} <(A°,\delta A_{\#}), \underline{C}_i > \in R \ \& \ R \in \text{CR- relations}$$

The set of all CR-relations that can be defined for an entity-region with respect to a *cover* represents the topological relationship between the *cover* and the entity-region. This set of relations specifies the location of the entity-region C within the *cover-structure.* Each of these sets of CR-relations corresponds to the equivalence class of all regions that have topologically equivalent relations to the same elements of the *cover-structure.* They have equivalent location within the *cover*. Consequently we have a n-to-1 mapping between entity-regions and CR-relation sets (location) with respect to a *cover-structure*.

4.4. Representation of the Location of Regions with Respect to Cover-structures

Now we apply the notion figure and ground (Talmy 1983) from Linguistics to entity regions and cover-structures. Entity regions are identified with the figures and a cover-structure is considered as a representation of the ground.
Let us now define a structure called figure-ground-position:

Definition 3 (figure-ground-position) :

A figure-ground-position is a triple ($(A°,\delta A_\#)$, $\underline{C}_i$, R) where

- $(A°,\delta A_\#)$ denotes the interior and a boundary segment of a region which belong to a cover,
- $\underline{C}_i$ denotes a connected intersection $\underline{C}_i \in C^*$ of an entity region C,
- R denotes the CR-relation which holds between the region-parts denoted by $(A°,\delta A_\#)$ and $\underline{C}_i$:

$$<(A°,\delta A_\#), \underline{C}_i> \in R \text{ , } R \in \text{ CR- relations;}$$

and sets of figure-ground-positions, called figure-ground-location:

Definition 4 (figure-ground-location) :

The set $LOC_{cover}(C)$ of figure-ground-positions ($(A°,\delta A_\#)$, $\underline{C}_i$, R) between a cover of regions (*cover*) and a entity-region C , that has the following properties :

$$LOC_{cover}(C) = \{ ((A°,\delta A_\#), \underline{C}_i, R) \mid (A°,\delta A_\#) \in \textit{cover-structure} \ \& \ \underline{C}_i \in C^* \ \& \ R \neq \text{CR-empty} \}$$

is called figure-ground-location of the region C within the *cover*.

If the identity of the *cover* is clear the figure-ground-location of the entity-region C with respect to the *cover* is denoted by LOC(C).

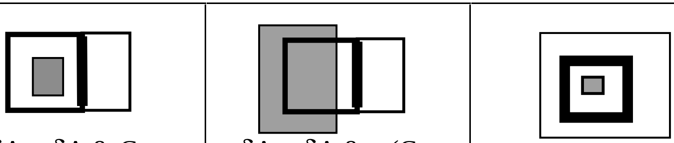
4.5. Binary Topological Relations and CR- Relations

From a single CR-relation between a pair $(A°,\delta A_\#)$ and a connected intersection $\underline{C}_i \in C^*$ of a region C, $<(A°,\delta A_\#) , \underline{C}_i> \in R$, $R \in \{R0...R3,E0...E3, \text{CR-empty}\}$, in general no unique conclusions about the topological relations between the regions A and C can be drawn. The reason therefore is that only a segment of the boundary of the region A namely $\delta A_\#$ is considered. We have to split the region C in connected intersection sets. This has the advantage that the results we get are more specific with respect to the boundary segment explicitly considered. As a consequence the results we get are less specific with respect to the whole regions A and C. In the special case $\delta A_\# = \delta A$ we can handle C as a whole and we get the intersection table of the 4-intersection and thus the topological relations between two (non-holed) regions. This happens for example if the region A is a hole in the region B. C is assumed to be non-holed.

CR- intersection	**4-intersectiom**
$<(A°,\delta A_B), C> \in R0$	$<C,A> \in$ inside
$<(A°,\delta A_B), C> \in R1$	$<C,A> \in$ covered by
$<(A°,\delta A_B), C> \in R2$	$<A,C> \in$ overlap
$<(A°,\delta A_B), C> \in R3$	$<A,C> \in$ meet
$<(A°,\delta A_B), C> \in$ CR-empty	$<A,C> \in$ disjoint
$<(A°,\delta A_B), C> \in E0$	$<A,C> \in$ equal
$<(A°,\delta A_B), C> \in E1$	$<C,A> \in$ contains
$<(A°,\delta A_B), C> \in E2$	$<C,A> \in$ covers
$<(A°,\delta A_B), C> \notin E3$ (C non-holed)	does not exist

Table 2: Correspondence of CR-intersection and 4-intersection for the case $\delta A_\# = \delta A$

In the general case $\delta A_\# \subset \delta A$ we do not know whether or not C intersects the rest of the boundary $\delta A \backslash \delta A_\#$ because it is not represented in the pattern of a single CR-intersection. The three cases in Figure 3 are not distinguishable in terms of a single CR-relation.



$\delta A_B \subset \delta A$ & $C \subseteq (A \backslash (\delta A_B)$	$\delta A_B \subset \delta A$ & $\neg(C \subseteq (A \backslash (\delta A_B))$	$\delta A_B = \delta A$ & $C \subseteq A°$

Figure 3: Three configurations of an entity-region C ( ) and a cover-region A ( ) that are not distinguishable in terms of the CR-relation :
$<(A°,\delta A_B),C> \in R0$

In order to get unique results, we have to consider all boundary segments of the regions under consideration. If we want to calculate the topological relations between A and C, we have to consider the subset of figure-ground-positions of LOC(C), which elements refer to the interior of A. The topological relations between the cover-region A and the entity-region C can uniquely be derived from this set of figure-ground-positions (Bittner 1996)(Lemma 3).

4.6. Example

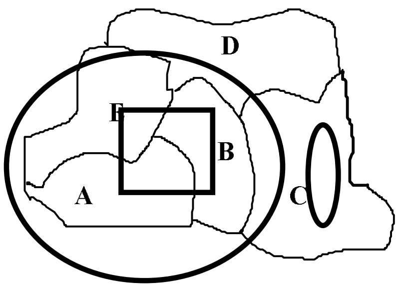


Figure 4: A *cover* of regions { A, B, C, D, E } and entity-regions , , and .

cover = { A, B, C, D, E }

cover-structure = { $(A°,\delta A_\varnothing)$, $(A°,\delta A_B)$, $(A°,\delta A_E)$, $(B°,\delta B_\varnothing)$, $(B°,\delta B_C)$, $(B°,\delta B_D)$, $(B°,\delta B_E)$, $(B°,\delta B_A)$, $(C°,\delta C_\varnothing)$, $(C°,\delta C_B)$, $(C°,\delta C_D)$, $(D°,\delta D_\varnothing)$, $(D°,\delta D_B)$, $(D°,\delta D_C)$, $(D°,\delta D_E)$, $(E°,\delta E_\varnothing)$, $(E°,\delta E_A)$, $(E°,\delta E_B)$, $(E°,\delta E_D)$ }

= F, LOC(F) = { $((A°,\delta A_E),F, R2)$, $((A°,\delta A_B),F, R2)$, $((A°,\delta A_\varnothing),F, R0)$, $((B°,\delta B_\varnothing), F, R0)$, $((B°,\delta B_C),F,R0)$, $((B°,\delta B_D),F,R0)$, $((B°,\delta B_E),F,R2)$, $((B°,\delta B_A),F,R2)$, $((E°,\delta E_D),F, R0)$, $((E°,\delta E_\varnothing),F, R0)$, $(E°,\delta E_A),F, R2)$, $((E°,\delta E_B),F, R2)$}

= G, LOC(G) = { $((C°,\delta C_B),G,R0)$, $((C°,\delta C_D),G,R0)$, $((C°,\delta C_\varnothing),G,R0)$ }

= H, LOC(H) = { $((A°,\delta A_\varnothing),H, E1)$, $((A°,\delta A_B),H, E1)$, $((A°,\delta A_E),H, E1)$, $((B°,\delta B_\varnothing),H, E1)$, $((B°,\delta B_C),H, E1)$, $((B°,\delta B_D),H, E1)$, $((B°,\delta B_E),H, E1)$, $((B°,\delta B_A),H, E1)$, $((C°,\delta C_\varnothing),H, R2)$, $((C°,\delta C_D),H, R2)$, $((C°,\delta C_B),H, E3)$, $((D°,\delta D_\varnothing),H, R0)$, $((D°,\delta D_E),H, R2)$, $((D°,\delta D_B),H, E3)$, $((D°,\delta D_C),H, R2)$, $((E°,\delta E_\varnothing),H, R2)$, $((E°,\delta E_A),H, E3)$, $((E°,\delta E_B),H, E3)$, $((E°,\delta E_D),H, R2)$ }

The *cover* in Figure 4 consists of the cover-regions A, B, C, D, E. This *cover* for example could correspond to the soil-structure of a particular environment. The *cover* is formally represented by the *cover-structure*. Each boundary segment is a representation of a boundary between two neighboring areas of different soil-types. Each interior refers to an area which soil-type is assumed to be homogenous with respect to a particular classification. One can see that there is redundant information in the set of figure-ground-positions. This is due to symmetric properties of some CR-relations.
F, G, and H correspond to phenomena conceptualized as entities. Let G be a lake, H a forest and F an area of pollution. H could also be element of a *cover* ‘vegetation-type’.
Each figure-ground-position within a figure-ground-location LOC() specifies the location of the entity represented with respect to one interior-boundarysegment-pair. From LOC(G) it can easily be derived

that the lake G is totally contained in the region corresponding to the area of soil type C. The forest-area H has parts that are outside the *cover* (e.g. $((A°,\delta A_{\varnothing}),H,E1) \in$ LOC(H)). For this parts we do not know the soil type. There are positions within the figure-ground-location of H that have the CR-relation E1: for example $((B°,\delta B_A),H, E1)$. This indicates that the boundary between soil type B and A runs somewhere inside the forest H. In $((B°,\delta B_A),H, E1)$ is further encoded that the boundary of the forest H does not cross the soil-area B. The forest H contains the soil-type-areas A and B: A *cover* region is contained in an entity-region if all figure-ground-positions of the figure-ground-location of the entity-region that refer to the *cover* region, contain a CR-relation that indicates that the whole boundary segment of the *cover* region is contained in the entity-region.
Both entity-regions G and H intersect the *cover* region C. Thus both figure-ground-locations contain figure-ground-positions that refer to the interior of the *cover* region C: $((C°,\delta C_B),H, E3)$ and $((C°,\delta C_B),G,R0)$. We cannot further specify the relations between G and H since no further information is available. It may be that the lake lies within the forest or outside the forest, or the edge of the forest runs along one bank of the lake, ...
From their figure-ground-locations we can derive that F and G must be disjoint and that F and H do intersect. The fact that F and G are disjoint implies that the pollution has not reached the lake yet. Whether or not it can reach it may depend on the soil-types B and C and the boundary between them.

5. Summary and Ongoing Work

The point of this paper is that a figure-ground-location formally and qualitatively characterizes the location of a geographic entity within its concrete environment. The emphasis on a concrete environment is due to the concern of geography with real spaces that exist on earth and are meaningful for human beings rather than on abstract spaces existing only in human minds. Since 'The Environment' as a whole can neither be conceptualized nor be processed on a computer, the field view of 'The Environment' under consideration is applied in order to extract relevant aspects and their spatial structure. In this particular approach we concentrate on problems where the application of the field view results in a regional partition of space. Since we further concentrate on natural phenomena, we only consider topological structural aspects. This is due to the indeterminate character of the boundaries established by the application of the field view of natural phenomena of space. Topology does not force us to specify unknown metric positions. We model the resulting structure by a set of regions that are either disjoint or meet each other and call it *cover*. We use point set topological notions of interior and boundary of a region to formalize *covers*. The formalization of a *cover* results in a *cover-structure*. A *cover-structure* consists of a set of interior-boundarysegment-pairs. A boundary segment is a connected subset of the boundary of a region that belongs to a *cover*. Each boundary segment is shared either with a neighboring *cover* region or with the complement of the whole *cover*.
A geographic entity is represented by a set of non-overlapping regions. For simplification in this formalization only single, none-holed regions are considered.
Location is the relation between a *cover* and an entity-region. It is formalized as a set of relations of parts of the region and parts of elements of the *cover* denoted by elements of the *cover-structure.* This set of relations is called figure-ground-location. The elements of a figure-ground-location are called figure-ground-positions. A figure-ground-position is a triple connecting an element of the *cover-structure* $(A°\delta A_{\#})$, an element of the connected intersection set of the entity-region C, $\underline{C}_i \in C^*$ and the CR-relation that holds between the sets that are referred by $(A°\delta A_{\#})$ and $\underline{C}_i$. The set of CR-relations covers the whole domain of topologically (in terms of the content of the intersection of subsets) distinguishable configurations that can occur between the interior of a region, a boundary segment of this region and a non-separated subset of another region.
The cover is finite. Thus the *cover-structure* is finite. The set of CR-relations is finite too. We use point set topology based on infinite abstract point sets only for the formal definition of the primitives and the relations between them. The resulting formal structures used for representation are finite sets and each element is directly related to an element of the conceptual model. On this level we only process sets of tuples representing *cover-structures* or representing relations between regions and *cover-structures*.
Thus we have a tuple-based representation of geographic space that takes the concrete topological structure of a particular environment under consideration into account. The qualitativeness of the representation is due to the definition of the CR-relations as topologically equivalent configurations.

Advantages of figure-ground-locations with respect to their application in GIS are :

- The representation consists of sets of tuples which can be easily stored in tables of relational database systems. Reasoning can be formalized in terms of operations on these sets of tuples and thus be transformed to relational algebra (Codd 1970) and implemented for example in SQL.
- Figure-ground-locations can be easily derived from geometric representations or from a small set of samples.
- Spatial and non-spatial data can be directly related to each other without reference to implicit geometric data structures.

A limit of figure-ground-locations is that they only preserve a subset of the topological relations between the entity-regions depending on the structure of the cover (example : relations between regions G and H cannot be specified).

Ongoing work will further formalize the conceptual model. Therefore topological relations between different *cover-structures* will be formalized. The composition of these relations and CR-relations will be used to transform figure-ground-locations between different covers. Furthermore operations on figure-ground-positions will be used to derive topological relations between the corresponding regions. This will be compared and combined with techniques based on relation composition.
If the structure of the cover follows a regular pattern (e.g. the pattern proposed in (Frank 1991) for direction reasoning) then topological reasoning and reasoning with cardinal direction becomes integrated in the same framework.

6. Acknowledges

I gratefully thank Andrew Frank for valuable comments and review of earlier versions of this paper.

7. References

Alexandroff, P. (1961). Elementary Concepts of Topology. New York, NY, Dover Publications.
Bittner, T. (1995). Levels of Spatial Inference, Department of Geoinformation., TU-Vienna
Bittner, T. (1996). A Formal Theory about Covers and Regions, Department of Geoinformation, TU-Vienna.
Codd, E. F. (1970). “A Relational Model of Data for Large Shared Data Banks.” CACM **13**(6): 377 - 387.
Couclelis, H. (1992). People Manipulate Objects (but Cultivate Fields): Beyond the Raster-Vector Debate in GIS. Theories and Methods of Spatio-Temporal Reasoning in Geographic Space, Springer.
Egenhofer, M. (1991). Reasoning about Binary Topological Relations. Advances in Spatial Databases, SSD, Springer.
Egenhofer, M. and J. R. Herring (1990). A mathematical Framework for the Definition of Topological Relationships. 4th International Symposium on Spatial Data Handling, Zurich, Switzerland, IGU.
Frank, A. U. (1991). Qualitative Spatial Reasoning about Cardinal Directions. Auto-Carto10, ACSM & ASPRS.
Frank, A. U. and W. Kuhn (1986). Cell Graphs: A Provable Correct Method for the Storage of Geometry. Second International Symposium on Spatial Data Handling, Seattle, WA, IGU.
Freksa, C. (1991). Qualitative Spatial reasoning. Cognitive and Linguistic Aspects of Geographic Space. D. Mark and A. U. Frank, Kluwer Academic Press.
Freksa, Chr. (1992). Using Orientation Information for Qualitative Spatial Reasoning. Theories and Methods of Spatio-Temporal Reasoning in Geographic Space. A. U. Frank, I. Campari and U. Formentini, Springer.
Hernandez, D. (1994). Qualitative Spatial Reasoning, Springer.
Laurini, R. and D. Thompson (1994). Fundamentals of Spatial Information Systems, Academic Press.
Mark, D. and A. Frank (1995). “Experiential and Formal Models of Geographic Space.” Environment and Planning.
Nunes, J. (1991). Geographic Space as a Set of Concrete Geographical Entities. Cognitive and Linguistic Aspects of Geographic Space. D. Mark and A. U. Frank, Kluwer Academic Press.

Spanier, E. (1966). Algebraic Topology. New York, McGraw-Hill Book Company.
Talmy, L. (1983). How Language Structures Space. Spatial Orientation: Theory, Research, and Application. H. Pick and L. Acredolo. New York, NY, Plenum Press.
Zubin, D. (1989). A model for scale differentiation of spatially distributed percepts. Languages of Spatial Relations: Initiative Two Specialist Meeting Report, NCGIA Technical Paper.